26[65K10]—*Numerical methods for least squares problems*, by Åke Björck, SIAM, Philadelphia, PA, 1996, xiii + 408 pp., $25\frac{1}{2}$ cm, softcover, $47.50

Conceptually, the easiest multi-dimensional optimization problem of all is the linear least squares problem. The problem is based on a linear model and the (unique) solution is easy to "write down" in the full rank case: it is the solution of the corresponding normal system of equations. Even in the rank-deficient case the solution set is well-understood and can be easily expressed in the language of the singular value decomposition. Therefore it might be surprising to learn that linear least squares problems (and many closely related problems) have considerable computational complexity. The business of numerically solving linear least squares problems is a serious one, requiring knowledge of a wide range of computational techniques.

"Numerical Methods for Least Squares Problems" by Åke Björck presents a modern "full-service" treatment of numerical methods for the linear least squares problem. This book is very well-written, comprehensive, and up-to-date. It is suitable for use as a textbook for a course on numerical methods for linear least squares problems as well as a reference for both practitioners and researchers.

The first two chapters cover the basic properties and factorizations: singular-value and QR decompositions, rounding error analysis, rank deficiency and ill-conditioning, and condition number estimation. A thorough treatment is given, though much of this material is covered in other modern texts on matrix computations, with minor variations in emphasis.

Efficiency in numerical methods is often obtained by exploiting the proximity of one problem instance to another. With cleverness much of the computational work in solving one instance can be amortized over several. In this setting "proximity" often means the defining matrices differ by a few rows or columns or, more generally, by a low-rank matrix. Chapter 3 discusses how to modify existing factorizations, in stable and efficient ways, to enable the efficient solution of proximal linear least squares problems.

The author does a wonderful service by including Chapter 4, a discussion of optimization problems closely related to least squares. In particular, I am very happy to see sections on l_1, l_p, l_∞, and robust linear regression (amongst others). Several modern numerical techniques for solving these less popular (but important!) norm-minimization problems are very similar indeed to the least squares machinery—this appealing aspect is recognized in this book.

Many practical least squares problems come with additional constraints on the variables. Chapter 5 discusses several numerical techniques for dealing with linear constraints, both equality and inequality. The chapter concludes with a short discussion of quadratically constrained least squares problems.

Chapters 6 and 7, concerned with large-scale problems, constitute the most important contribution of this book, in my opinion. Least squares problems that arise in serious applications are often very large systems, typically sparse. The robust and efficient methods discussed in the first five chapters become impractical, with respect to both space and time, in the large-scale setting. Chapters 6 and 7 discuss methods and techniques, direct and iterative, that are tailored to large (and sparse) problems. There is good detail here, especially with respect to sparse direct methods, and yet the big picture is never far from the surface. Many recent developments are discussed, briefly but informatively.

Finally, in chapters 8 and 9 the basic material is extended to problems with special bases (e.g., Toeplitz systems, Vandermonde matrices) and nonlinear problems.

In summary, this is a wonderful book, suitable as a text as well as a research reference. What is missing? The important bases are all touched upon, though two timely topics are given rather brief treatment: parallel methods and the surprising effectiveness of the (modified) normal system approach in interior point methods for linear programming. There is some discussion of the latter, but it is brief and (already) a bit out of date.

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27[**11–00**]—*Number-theoretic function products*, by R. G. Buschman, Buschman, Langlois, OR, 1996, vi +69 pp., 28 cm, vinyl cover, spiral bound, $12.50

In this work, the author has collected, for number-theoretic functions, properties for various products [Dirichlet, integer, lcm (Lehmer), Max, unitary, exponential, integral convolution]. Included are lists of specific products, multiple factor products, alternative factorizations (summation identities), and various inversion formulas.

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